An Experimental Investigation on the Performance of Priority Rules for the Dynamic Stochastic Resource Constrained Multi-Project Scheduling Problem

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1 Introduction

Project-based organizations such as R&D, are often matrix organizations with numerous functional departments. Projects arrive in a stochastic manner and their activities having typically stochastic durations are processed by the resources available in the functional departments. As a consequence projects suffer from delay and missed due dates. A common method for approaching these real-life complicated conditions is via implementing a dynamic scheduling policy using a priority rules for specifying the order of processing of activities by a resource. Despite the practical relevance of this dynamic stochastic resource constrained multi-project scheduling problem (DSRCMPSP) only limited research has been undertaken so far to address it. A major part of the literature related to the multi-project scheduling problem is dedicated to the static and deterministic case with the set of projects given at the outset and all parameters such as activity durations known in advance. Thus, the objective of our research is to provide a comprehensive comparison of well-known priority rules, which have been proposed for simpler settings for the weighted tardiness objective and to adapt them as needed for application to the more practically relevant DSRCMPSP. From the literature, we identify a number of priority policies which have shown good performance in prior computational studies. Whereas all these rules have shown promising results for the weighted tardiness objective they have never been compared together in the same study nor in the dynamic stochastic resource constrained setting. In order to introduce the problem formally, we follow the framework of (Adler *et. al.* 1995) and depict an organization as a set $\mathcal{R}$ of resources. Resource $r \in \mathcal{R}$ comprises c_r identical units, each capable of processing one activity at a time. Projects arrive dynamically according to a stochastic arrival process. We consider a stream of projects $j = 1, \ldots, J$ where each project j arriving at time a_j is of type $p_j \in \mathcal{P}$ and is assigned a due date D_j. The idea of project types reflects the fact that often projects have structure in common such as new development projects or reformulation projects. Project type $p \in \mathcal{P}$ has a weight w_p, an interarrival rate λ_p and is comprised of a set of activities $\mathcal{V}_p$ and a set of precedence relations $\mathcal{A}_p$ of the type finish-to-start with minimum time lag of 0. Each precedence relation between activity i and i' is written as a tuple $(i, i') \in \mathcal{A}_p$. In order to be processed, activity $i \in \mathcal{V}_p$ seizes one unit of resource $r_{ip} \in \mathcal{R}$ for a stochastic duration d_i with mean $\overline{d}_{ip}$. $\mathcal{E}_r(t)$ refers to the set of activities being processed by resource r at time t where each activity i of project j is referred to by tuple (i, j). Define the random variable C_{ij} as the realized completion time for activity i of project j and C_j the realized completion date of the project with $C_j = \max_{i \in \mathcal{V}_j} C_{ij}$. The objective is to minimize the long term expected weighted project tardiness with J being the total number of projects arriving at the system:

$$\text{Min}Z = \lim_{J\to\infty} E\left[\frac{1}{J}\sum_{j=1}^{J} w_j(0, C_j - D_j)^+\right] \tag{1}$$

subject to the following constraints

$$a_j + d_{ij} \leq C_{ij} \quad \forall j = 1, \ldots, J; i \in \mathcal{V}_j \tag{2}$$
$$C_{ij} + d_{i',j} \leq C_{i',j} \quad \forall j = 1, \ldots, J; (i, i') \in \mathcal{A}_{p_j} \tag{3}$$
$$|\mathcal{E}_r(t)| \leq c_r \quad \forall r \in \mathcal{R}, t \geq 0 \tag{4}$$
$$C_j \geq C_{ij} \quad \forall j = 1, \ldots, J, i \in \mathcal{V}_j \tag{5}$$

The objective function (1) minimizes the long term expected weighted project tardiness Z where J denotes the number of arrived projects and $(x)^+$ is $\max(0, x)$. Constraints (2) forces each activity i of a project j not to start before the project has arrived at a_j. Constraints (3) specify the precedence constraints between the activities of each project j. Constraints (4) depict the resource constraints: At any point in time t the number of activities that are in process by resources r, $|\mathcal{E}_r(t)|$, must not be greater than the number of available units, c_r of that resources. Constraint (5) defines the project completion time for project j.

2 Priority rules and experimental design

Rules addressing the weighted tardiness objective typically combine information about the input parameters: due date, activity processing time, and weight. There two main approaches according to which the pieces of information are combined. The first approach is to consider some ratio involving due dates and processing times. A second approach is to consider a time-sensitive binary switch in emphasis from due date to processing time. Thus, we distinguish between ratio rules and binary switch rules. Probably the first evidence of the "ratio" approach is found in the work of (Carroll 1965) and his so-called "c over t" dispatching rule which was inspirational for a series of work by (Lawrence and Morton 1993), and (Morton and Pentico 1993). For binary switch rules the seminal work is provided by (Baker and Bertrand 1982) with their "modified due date" priority rule and later further developed by (Baker and Kanet 1983), (Dumond and Mabert 1988), (Anderson and Nyirenda 1990), and (Kanet and Li 2004).

The following ratio rules have been considered:

- BD with Myopic Activity Costing (BD-MC)
- BD with Global Activity Costing and Uniform Resource Pricing (BD-GC-U)
- BD with Global Activity Costing and Dynamic Resource Pricing (BD-GC-D)

The major rules falling into the category of binary switching rules include the various forms of the "modified due date" approach first provided by Baker and Bertrand (Baker and Bertrand 1982). We study several variations of the modified due date approach applying it to the DSRCMPSP.

- Weighted Modified Due Date (WMDD)
- Weighted Modified Operation Due Date (WMOD)
- Weighted Critical Ratio and Shortest Processing Time (W(CR+SPT))
- Weighted Critical Ratio and Global Shortest Processing Time (W(CR+GSPT))
- Weighted Due Date Modified Shortest Activity from Shortest Project (WSASP-DD)

Finally, for benchmarking purposes we have added some simple but well known rules. They have been selected as they are frequently used in related studies of multi-project and job shop scheduling problems:

- First-Come, First-Serve (FCFS)
- Weighted Minimum Slack (WMINSLK)
- Weighted Shortest Processing Time (WSPT)
- Random (RAN)

For our study we created a number of problem instances with various parameters being controlled. In brackets the values used are indicated.

- α_p (20%,80%): Percentage of tardy projects of type p when using the RAN scheduling policy. This implicitly controls due date tightness.
- $\mathcal{R}$ (1, 3, 5, 10, 15, 20): Number of resources.
- ρ (0.7, 0.9): Resource utilization.
- $\text{CV}^{\overline{d}}$ ($[0, 0.4]$, $[0.8, 1]$): Coefficient of variation of the expected durations belonging to the activities processed by each resource.
- OS (0, 0.2, 0.4, 0.6, 0.8, 1.0): Order strength of the project networks.

In each problem instance we have two project types ($p \in \{1, 2\}$) having two different networks respectively but with the same order strength and number of activities (20). Finally, we obtained $1,152$ instances. For each problem instance we simulate each of the 11 priority policies in a simulation run which results in $12,672$ runs.

3 Summary of the results

In the following we provide the results of the simulation study. The performance of a rule is measured as the normalized weighted tardiness $Z^n(\pi)$ given by $Z^n(\pi) = Z(\pi)/Z(\pi_{\text{RAN}})$. Duncan tests were undertaken to detect groups of rules where rules in different groups show a significant difference in performance while rules in the same group do not. Table 1 provides the performance of all priority rules as well as the group a rules belongs to, according to the Duncan test. W(CR+SPT) is the best rule being significantly superior to all other rules. This extends the findings of (Kutanoglu and Sabuncuoglu 1999) for the dynamic job shop problem.

Rule	$\overline{Z}^n(\pi)$	Group
W(CR+SPT)	0.31	g
BD-GC-D	0.44	f
BC-MC	0.46	f
WMOD	0.51	f
WMINSLK	0.62	e
FCFS	0.63	e
WSASP-DD	0.81	d
BD-GC-U	0.95	c
WSPT	0.98	bc
W(CR+GSPT)	1.07	ab
WMDD	1.11	a

Table 1. Overall performance of the priority rules and group according to the Duncan test

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