Metric Estimations for a Resource Leveling Problem With Variable Job Duration

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1 Introduction

We consider a Resource Leveling Problem (RLP), which remains an object of intensive research in project scheduling. It is possible to apply this formulation for many real-life cases, such as transportation, production, education, and management. It is known to be NP-hard in the strong sense, see Neumann et al. (2002). A set of project jobs must be scheduled in a planning horizon without preemption, jobs require a different amount of resources, and resource capacity is given during the planning horizon. There are also precedence relations between jobs. For the RLP, three basic types of the objective function are known in the literature: the total squared resource utilization cost, total overload cost, and the total adjustment cost (see Rieck & Zimmermann (2015)).

Usually, these basic formulations are enriched with new features and constraints to make the model closer to a particular practical case. Hans (2001) presented a branch-and-price approach for a Resource Leveling Problem which allows jobs to be implemented non-uniformly, i.e. jobs have different intensities in different moments (discrete-time periods). Kis (2005) constructed the branch and cut algorithm for this model which outperformed branch and price approach. We focus on the total overload cost objective, a weighted sum of resource amount which is used in addition to given available resource amount to fit in the project deadline constraint. This case was studied by Bianco et al. (2016) and Baydoun et al. (2016). Bianco et al. (2016) presented a solution approach for the model with generalized precedence relations with time lags (GPR). Baydoun et al. (2016) review the case when the partial intersection of predecessor and successor job is allowed. To the best of our knowledge, these formulations are the closest to our model. In this paper we study a generalized version of the RLP. It was introduced earlier (see Tarasov et al. (2019)) with a comparison of solution quality to the concepts applied by Bianco et al. (2016) and Baydoun et al. (2016).

2 Problem formulation

Our problem planning horizon is divided into $T = \{1, ..., m\}$ periods of given length d. All jobs have their minimal duration superior to d. There is a fixed deadline for all jobs which is the end of the planning horizon. Available workload for each resource type $r \in R$ in each period $t \in T$ is limited by the free work volume L_{rt}. Additional workload for this resource type $r \in R$ costs e_r. The duration of the job is not given and can also vary in some limits, depending on the total workload required to implement this job by different resources. For each job $j \in J$ required workload W_{jr} for each resource $r \in R$ is given, with upper and lower bounds on assigned workload per period defined by multipliers, $p_{min,jr}$ and $p_{max,jr}$, $r \in R$. If some job $j \in J$ is implemented during time d_{jt} in period $t \in T$, assigned workload has lower and upper bounds $p_{min,jr}d_{jt}$ and $p_{max,jr}d_{jt}$, respectively. Precedence relations are represented in a directed graph which should be acyclic. We present all problem instance parameters and decision variables in Table 1, and the MILP model from Tarasov et al. (2019) below.

Table 1. Problem instance parameters and decision variables

Parameters	
T	planning horizon, $T = \{1, ..., m\}$
d	period length
R	resources set
L_{rt}	available workload of resource $r \in R$ in period $t \in T$
e_r	extra resource cost
J	jobs set
W_{jr}	job $j \in J$ workload on the resource $r \in R$
$p_{min,jr}$	job $j \in J$ minimum assigned workload of resource $r \in R$
$p_{max,jr}$	job $j \in J$ maximum assigned workload of resource $r \in R$
P	the set of arcs in precedence graph
Decision variables	
S_{jt}	binary step : equals 1 if job $j \in J$ starts in $\forall t_1 \in T$, $t_1 \leq t$, 0 otherwise
E_{jt}	binary step : equals 1 if job $j \in J$ ends in $\forall t_1 \in T$, $t_1 < t$, 0 otherwise
d_{jt}	duration of job $j \in J$ in period $t \in T$
c_{jrt}	work volume of the job $j \in J$ on the resource $r \in R$ in period $t \in T$
o_{rt}	extra cost of the resource $r \in R$ in period $t \in T$

$$S_{jt} \geq E_{jt},\ S_{jt} \leq S_{j,t+1},\ E_{jt} \leq E_{j,t+1},\ \forall j \in J,\ \forall t \in T; \tag{1}$$

$$d_{jt} \leq d\ (S_{jt} - E_{jt}),\ \forall j \in J,\ \forall t \in T. \tag{2}$$

$$d_{jt} \geq d\ (S_{jt} + S_{j,t-1} - 1 - E_{jt} - E_{j,t+1}),\ \forall j \in J,\ \forall t \in T. \tag{3}$$

$$S_{j_2 t} \leq E_{j_1,t+1},\ d_{j_1 t} + d_{j_2 t} \leq d,\ \forall t \in T,\ \forall (j_1, j_2) \in P; \tag{4}$$

$$d_{j_1 t} + d_{j_2 t} \leq d,\ \forall t \in T,\ \forall (j_1, j_2) \in P. \tag{5}$$

$$p_{min,jr} d_{jt} \leq c_{jrt} \leq p_{max,jr} d_{jt},\ \forall j \in J,\ \forall r \in R,\ \forall t \in T; \tag{6}$$

$$\sum_{t \in T} c_{jrt} = W_{jr},\ \forall j \in J,\ \forall r \in R; \tag{7}$$

$$o_{rt} \geq e_r (\sum_{j \in J} c_{jrt} - L_{rt}),\ \forall t \in T,\ \forall r \in R; \tag{8}$$

The objective function of our problem is represented in the following form: $\sum\limits_{r \in R} \sum\limits_{t \in T} o_{rt}$.

3 Metric approach in scheduling

In this paper, we study the scheduling problem solution approach, which was presented by Lazarev (2009). The latter approach has been shown to be effective in dealing with some NP-hard scheduling problems, including the total tardiness minimization problem Lazarev et al. (2017) and the single-machine scheduling problem Lazarev & Kvaratskheliya (2010).

The idea of this approach is to use polynomially solvable subcases of the problem, which is NP-hard in general. It allows producing the solution for an arbitrary instance with a guaranteed accuracy (objective function value difference) in polynomial time. Any scheduling problem input data instance represents the point in $f(n)$-dimensional space Ω, where n is the number of jobs in the instance. Firstly, if the same schedule π (the permutation of jobs) is used as the solution for two different arbitrary instances A and B, it is possible to make the estimation of the difference between objective function values $V_A(\pi)$ and $V_B(\pi)$ for these instances. This estimation is formed as the metric $\rho(A, B)$ defined on $\Omega \times \Omega$, such that

$$|V_A(\pi) - V_B(\pi)| \leq \rho(A, B).$$

Secondly, suppose there are two schedules π^A and π^B that are the optimal solutions for instances A and B, respectively. It also possible to construct the estimation for the expression

$$V_A(\pi^B) - V_A(\pi^A) \leq \Delta(\rho(A, B))$$

and prove that it depends on $\rho(A, B)$. Then $\Delta(\rho(A, B))$ is the absolute accuracy for the case when π^B is used as the solution for instance A instead of the real optimal solution. The original idea is to use the schedule π^B that is optimal for some polynomially solvable instance B as the solution for the original problem instance A, such that the difference between the objective function values is minimum, i.e., the value of $\rho(A, B)$ is minimum.

4 Application to a Resource Leveling Problem

Although in the case of a Resource Leveling Problem it is difficult to specify the solvable subclasses, it is possible to get the estimations. Our problem provides more possible options to construct the proper estimations $\rho(A, B)$. Therefore, it allows to make the following steps:

1. the construction of a metric function $\rho(A, B)$ for arbitrary instances A and B in case of particular model
   - present model implies some feasibility criteria on the solution π to be used for particular instance A;
   - we provide the objective function value difference estimations for instances A and B if the difference is only in some particular parameter (e.g. L_{rt}, W_{jr} etc.);

   Lemma 1. *An example for L_{rt}. Consider the instances A and B, which are different only in the parameters L_{rt}. If we apply the same solution σ to the both instances, the upper bound for objective function values difference is*

   $$|V^A(\sigma) - V^B(\sigma)| \leq \rho_L(A, B) = \sum_{r \in R} e_r \sum_{t \in T} |L^A_{rt} - L^B_{rt}|. \quad (9)$$

   Proof.
   $$|V^A(\sigma) - V^B(\sigma)| = |\sum_{r \in R} \sum_{t \in T} o^A_{rt} - \sum_{r \in R} \sum_{t \in T} o^B_{rt}|, \quad (10)$$

   here $o_{rt} = \max\{0, e_r(\sum_{j \in J} c_{jrt} - L_{rt})\}$, and taking into account that costs are equal $e^A_r = e^B_r = e_r$ and $|\max\{a, b\} - \max\{c, d\}| \leq \max\{|a - c|, |b - d|\}$,

   $$|V^A(\sigma) - V^B(\sigma)| \leq \sum_{r \in R} \sum_{t \in T} |e^A_r(\sum_{j \in J} c_{jrt} - L^A_{rt}) - e^B_r(\sum_{j \in J} c_{jrt} - L^B_{rt})| \leq$$
   $$\leq \sum_{r \in R} e_r \sum_{t \in T} |L^A_{rt} - L^B_{rt}|.$$

   □

- we consider the objective function value difference estimations when several parameter types vary from instance A to B;
- it is important to prove that general case estimations can be combined from the particular parameter changes and the impact is not multiplied;

2. the formulation of estimations $V_A(\pi^B) - V_A(\pi^A) \leq \Delta(\rho(A, B))$ of the difference between objective function values of optimal solutions for arbitrary instances A and B.

5 Conclusion

To sum up, we propose to apply the metrization approach to a Resource Leveling Problem. This approach allows studying the error upper bound when the given optimal solution of the instance is used as a suboptimal solution for another instance (which may differ in some parameters). The general idea is to provide theoretical estimations of the guaranteed absolute accuracy in this case and use these properties to deal with some real-life issues, for example, data uncertainty.

Future work includes the detailed study of possible improvements of these estimations and the allocation of useful subclasses of instances that are solved in a reasonable time. The second goal is to study useful applications to uncertainty cases and numerical experiments, for example:

- describe the conditions when there are some changes in the instance data, but the same solution (or the scheduling part, in particular) still remains optimal;
- provide the proven accuracy solutions for the cases with uncertain data in some parameters (e.g. presented by ranges).

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