

# A Conjunctive-disjunctive Graph Modeling Approach for Job-Shop Scheduling Problem with Changing Modes

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## 1. Introduction

Reconfigurable Manufacturing Systems (RMS) have been defined in (Mehrabi et al., 2000) as an effective approach to deal with unpredictable and high-frequency market changes that are facing industries. To cope with such changes, the production systems must be adaptive and able to evolve in order to consider 1) changes in parts of existing products; 2) fluctuations in demands; 3) evolution in legal regulations and 4) evolution in process technology. Meanwhile, the scheduled operations remain partially manual like material handling, carrying and processing jobs as stressed by (Napolitano, 2012). The assignment of operators to operations must include personal skills, training and experience in order to match the competences and/or functionalities required by the operations to be performed (Ferjani et al., 2017; Grosse et al., 2015). In RMS, the sequential execution of operations may depend on the job operation sequence that can refer to Flow-shop, Job-shop, etc. Including flexibility for processing operations remains possible at each step of the job-sequence. Meanwhile, reconfigurability is the capacity of a set of machines to be reconfigured in a period of time, which can be seen as setup times. Machine activation delay may include cleaning the working zone, loading, positioning and unloading the parts (jobs) and can imply costs coming from energy expenditures, equipment maintenance and labor as stressed by (Borgia et al., 2013). Hence, in RMS a solution is composed by a set of configurations applied sequentially and thus a sequence-dependent processing time of operations and sequence dependent setup times have to be considered in such a production system. If sequence depend setup times are features of several research projects in the scheduling community as stressed in (Sharma and Jain, 2016; Shen et al., 2018), these works generally consider setup times at the operation level, whereas several modifications of the system may occur in RMS requiring several resources to be inactive during reconfigurations.

Hence, the problem addressed in this research project is different from the one introduced in (Essafi et al., 2012) since it does not encompass design and line balancing but only machine operations, and is concerned with makespan minimization and not minimization of the cost of the line. Actually, the problem is closer to the former vision provided by (Liles and Huff, 1990) who first indicated the necessity to schedule efficiently operations in reconfigurable manufacturing environments. As stressed by (Azab and Naderi, 2015) very few papers deal with scheduling in RMS. In their research work, they addressed reconfigurations in the context of Flow-shop production systems, but they did not investigate graph modelling.

The present paper is dedicated to scheduling in reconfigurable manufacturing systems where operators assignment to machine allows to define several modes meaning that processing time of operations is varying according to chosen configurations. The work specifically focuses on the graph modelling of the problem in the context of a Job-shop-like production system and introduces encoding and decoding of solutions.

## 2. Graph modelling and representation of solutions

The problem under study is stated as a reconfigurable job-shop manufacturing system where a set  $J$  of  $n$  jobs has to be scheduled  $J = \{J_1, J_2 \dots J_n\}$  on a set  $M = \{M_1, \dots, M_m\}$  of  $m$  machines. Each job in  $J$  consists in a set of operations  $O_j = \{O_{1j}, \dots, O_{mj}\}$ . The whole system operates under configurations. Moving from a configuration to another may affect specific machines, resulting in variations in processing times of operations. Hence, each operation  $O_{ij}$  has a processing time  $P_{ij}^k$  where  $k$  denotes the chosen configuration. Configuration differs from setup times, since transition between configurations can affect several machines and configurations can be activated only when these machines are inactive. Considering two configurations  $k_1$  and  $k_2$ , identifying machines that are concerned by a switch from configuration  $k_1$  to  $k_2$  is achieved through vectors  $R_{k_1, k_2}^{M_u}$ , where each value of the vector is valued 0 if the machine  $M_u$  is not concerned with transition, 1 otherwise. A reconfiguration time  $T_{k_1, k_2}$  is required when switching from a configuration  $k_1$  to  $k_2$ . The objective is to schedule efficiently operations and to define configuration assignments in order to minimize the completion time of all operations (makespan). In the following, the data bellow are considered, where  $M_u(P_{ij}^k)$  denotes the processing time on machine  $M_u$  according to configurations.

Table 1. processing times of operations in configuration 1

Product	$O_{1j}$	$O_{2j}$	$O_{3j}$
$j = 1$	$M_1(10)$	$M_2(6)$	$M_3(17)$
$j = 2$	$M_2(15)$	$M_1(10)$	$M_3(20)$
$j = 3$	$M_3(4)$	$M_2(10)$	$M_1(20)$

Table 2. processing times of operations in configuration 2

Product	$O_{1j}$	$O_{2j}$	$O_{3j}$
$j = 1$	$M_1(13)$	$M_2(4)$	$M_3(12)$
$j = 2$	$M_2(12)$	$M_1(17)$	$M_3(23)$
$j = 3$	$M_3(7)$	$M_2(16)$	$M_1(10)$

Table 3. Definition of  $R_{k_1, k_2}^{M_u}$

Configurations	$k_1$	$k_2$
$k_1$		(1;1;1)
$k_2$	(1;1;1)	

Tables 1, 2 and 3 introduce data of a 3 jobs, 3 machines Job-shop Scheduling Problem, where processing times of operations depend on configurations. As can be seen in Table 1 and 2, processing times of operations on machine  $M_1$  are different whether configurations  $k_1$  or  $k_2$  are selected. Assignment of machines when switching from a configuration to another is introduced in Table 3. In this problem, all machines are affected by a change in configuration, and hence, they must be all inactive when switching from a configuration to another and without of generality the reconfiguration time  $T_{k_1, k_2}$  is set to 1 time unit.

In scheduling problems it is classical to use a conjunctive-disjunctive graph approach that have been proved to be efficient by (Roy and Sussmann, 1964). For the incumbent problem, a conjunctive-disjunctive graph  $G(V, A, E)$  is considered where  $V$  corresponds to the operations,  $A$  denotes the arcs and  $E$  defines the edges. Initial arcs correspond to precedencies in jobs sequence of operations (i.e. an arc  $(O_{ij}, O_{i, j+1})$  exists in  $G$  between two successive operations of  $i$ ).  $E$  refers to edges that have to be oriented and initially contains edges relevant to operations that have to be processed on the same machines, and all edges that refer to configurations. An edge is considered between operations  $O_{ij}$  and  $O_{kj}$  if they can be processed in two different configurations that are impacting processing times of both operations. Similarly to (Dauzère-Pérès and Paulli, 1997) for the Flexible Job-shop, different shape lines can connect operations in order to distinguish configuration switches and machine disjunctions. The objective is to assign a configuration to each operation and to defined edges that connect operations using the same machine. The Figure 1 gives an example of a conjunctive-disjunctive graph after choosing configurations for operations.

For sake of clarity, two graphs are presented in Figure 1, the first one (A) concerns edges related to machine disjunctions (dashed lines), and the second one (B) displays edges related to configuration switches (dotted lines). In this figure, operations modeled with grey nodes are processed into configuration 1, and the ones with white nodes are processed into configuration 2. As operations of a given job are ordered, edges connecting two operations with different assigned configurations can be removed (i.e. edge between  $(M_1; M_3)$  on job  $J_1$  is useless) when other operation are present between them.

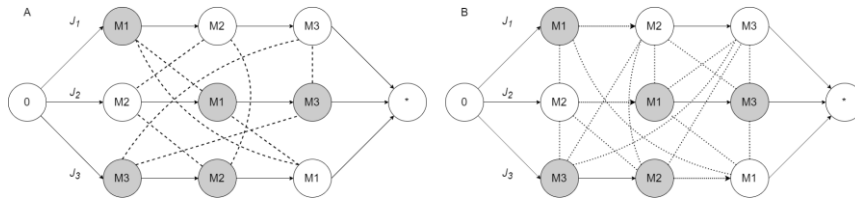


Figure 1. Graph A with edges for machine disjunctions and graph B with configuration disjunctions.

Modeling solutions is an important preliminary step before defining complex operators such as metaheuristics or local search. Indirect representations are widely spread in literature for scheduling problems (Cheng et al., 1996). For the incumbent problem two vectors are used. The first one ( $R$ ) is a vector by repetition (Bierwirth et al., 1996) which is an ordered list of job numbers (a job numbers is in the list  $m$  times with  $m$  the number of machines) and each occurrence of a job corresponds to one of its operations. The second vector ( $C$ ) is the configuration vector which is a list of configurations under which operations are processed. Both vectors represent a solution which is an orientation of all arcs (Fig. 2) considering  $R = [1; 2; 2; 3; 3; 1; 2; 1; 3]$  and  $C = [1; 2; 1; 1; 1; 2; 1; 2; 2]$ . Considering these vectors, defining a solution consists in reading the vectors from left to right applying an extension of the Bierwirth vector rules for graph generation. Figure 2 shows the evaluated graph after execution of one longest path algorithm.

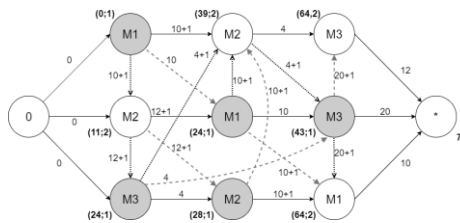


Figure 2. Evaluated conjunctive graph

In Figure 2, dashed arrows define sequence of operations on machines, while dotted arrows define reconfiguration switches. Each arc modeling reconfiguration switches are valued  $(P_{ij}^k + T_{kk'})$ . Starting time and configuration of operations are in bold in the figure. According to the vector  $R$ , the first operation scheduled is the first operation of job  $J_1$  and according to the vector  $C$ , it is processed with configuration 1, hence its processing time is 10 according to Table 1. The second operation in vector  $R$  is the first of job  $J_2$  processed on  $M_2$  with configuration 2, hence a reconfiguration switch occurs after operation  $O_{11}$  that required a 1 time unit of reconfiguration that delay the operation  $O_{12}$  starting time at 11. The third scheduled operation is  $O_{22}$  on machine  $M_1$  and configuration 1 and a reconfiguration occurs after  $O_{12}$ , and  $O_{22}$  will start at 24 (ending time of  $O_{12}$  plus reconfiguration time). The fourth scheduled operation is  $O_{13}$  that is the first operation on  $M_3$ , also processed with configuration 1, and hence, it starts after the last operation that affected  $M_3$ , with configuration 2. This process iterates until the end of both vectors  $R$  and  $C$ . The obtained Gantt chart is given in Figure 3.

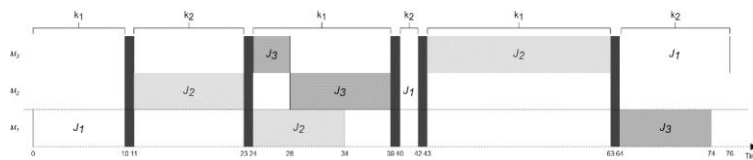


Figure 3. Gantt chart corresponding to evaluated Graph

As stressed on Figure 3, 5 reconfigurations are operated along the time horizon and they are respectively scheduled at times  $[10;11]$ ,  $[23;24]$ ,  $[39;40]$ ,  $[42;43]$  and  $[63;64]$ . As all machines are affected by these reconfigurations, it is not possible to schedule operations earlier, considering the given vectors  $R$  and  $C$ .

The Gantt of figure 3 does not define an optimal solution and could be further improved by local search operator for example. Future research is now directed on the design of a metaheuristic

that will consider the encoding vectors including specific operator such as construction heuristics, neighborhoods and local search operators. An effective local search approach should rely on an exploration of the critical paths that must create operator on the vector by changing order of operations in vector  $R$ , changing configurations in vector  $C$ , or both.

### 3. Conclusion

This work is at the corner stone of both scheduling and reconfigurable manufacturing systems communities since reconfigurations and setup times are very similar notions that are closed to the flexible terminology used in scheduling. In this research project, the Job-shop is extended with reconfiguration schemes. When a reconfiguration occurs, specific machines are affected and have to be stopped in order to apply the new configuration to the production system. It is possible to address small-scale instances using linear solvers but medium and large-scale instances remain intractable. The use of metaheuristics seems appropriate and will concern the upcoming research prospects. To this purpose, a conjunctive-disjunctive graph model is proposed. Adjoined with proper representation of solutions it is possible to map an element from the coding space with the proposed graph model through a decoding procedure. Two vectors are used to represent orientations of arcs and selection of configurations in the graph. In addition with metaheuristics, local search procedures relying on critical path exploration are currently investigated.

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