Multi-Scenario Scheduling with Rejection Option to Minimize the Makespan Criterion

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1. Introduction and problem definition

We study a set of single-machine scheduling problems, where job-processing times are uncertain at the time-point at which the scheduler has to make his scheduling decisions. We assume that there is a finite set of different scenarios that can affect the processing environment, and thus processing times are scenario-dependent. Scheduling problems with scenario-dependent processing times are mainly studied in the literature under the assumption that all jobs have to be scheduled in shop (see, e.g., Daniels and Kouvelis 1995; Yang and Yu 2002; Aloulou and Croce 2008; Mastrolilli et al. 2013; Choi and Chung 2016; and Kasperski and Zielinski 2016), exposing the manufacturer to a great deal of uncertainty (risk). The common way to control risk in the multi-scenario scheduling literature is to find a robust schedule, which minimizes the maximal value of the scheduling criterion between all scenarios (see, e.g., Daniels and Kouvelis 1995; Yang and Yu 2002; Aloulou and Croce 2008; Mastrolilli et al. 2013; and Kasperski and Zielinski 2016). A different approach to control risk is to reject the processing of some jobs by either outsourcing them or rejecting them altogether.

Although scheduling with rejection and multi-scenario scheduling are two solid fields in the scheduling literature, only Choi and Chung, (2016) consider these two approaches simultaneously. We aim to extend the relevant literature on multi-scenario scheduling with rejection in order to provide the manufacturer tools to coordinate outsourcing with scheduling decision in an uncertain processing environment. In this paper, we focus on single-machine problems with the objective of minimizing the makespan.

The set of problems we study is formally defined as follows: we are given a set $\boldsymbol{J} = \{J_1, J_2, \ldots, J_n\}$ of n independent, non-preemptive jobs that are available for processing at time zero. There is a set of q different scenarios, each of which defines a different possible set of job processing times (we consider both cases where q is a constant and an arbitrary value). By $p_j^{(i)}$ we denote the processing time of job J_j $(j = 1, \ldots, n)$ on the single machine under scenario i $(i = 1, \ldots, q)$. Moreover, by e_j we denote the cost of rejecting job J_j (e.g., outsourcing its processing to a subcontractor).

A solution π=(τ, $\sigma(\boldsymbol{A})$) is defined by (i) a partition $\tau = \boldsymbol{A} \cup \widehat{\boldsymbol{A}}$ of set $\boldsymbol{J}$ into two disjoint subsets $\boldsymbol{A}$ and $\widehat{\boldsymbol{A}}$ referring to the set of accepted and rejected jobs, respectively; and by (ii) a schedule $\sigma(\boldsymbol{A})$ of the accepted jobs (jobs in set $\boldsymbol{A}$) on the machine. Given a solution let

$$RC = RC(\widehat{\boldsymbol{A}}) = \sum_{J_j \in \widehat{\boldsymbol{A}}} e_j$$

be the total rejection cost. Moreover, let $C_j^{(i)}$ be the completion time of any job $J_j \in \boldsymbol{A}$ on the single machine under scenario i $(i = 1, \ldots, q)$. For a given scheduling criterion G, let $G^{(i)}$ be its value under scenario i $(i = 1, \ldots, q)$, and let $F^{(i)} = G^{(i)} + RC$ for $i = 1, \ldots, q$. We measure the quality of a solution by the following set of q different solution values

$$\mathbf{SV} = \left\{F^{(1)}, \ldots, F^{(q)}\right\}.$$

In this paper we consider the case where G is the makespan criterion, accordingly we have that

$$G^{(i)} = C_{max}^{(i)}(\boldsymbol{A}) = \max_{J_j \in \boldsymbol{A}} \left\{C_j^{(i)}\right\}.$$

As we measure the quality of any solution by q different solution values, many different problem variations can be considered (see, e.g., Gilenson et al. 2018). In this paper, we focus on the following problem variations, which are commonly analysed in the multi-criteria literature:

- Problem Variation 1 (PV_1): Given a set of non-negative parameters, $\theta^{(i)}$ $(i = 1,\ldots,q)$, find a solution, π, that minimizes the linear combination of $F^{(1)},\ldots,F^{(q)}$, i.e., that minimizes $\sum_{i=1}^{q}\theta^{(i)} F^{(i)}$.
- Problem Variation 2 (PV_2): Find a solution, π, that minimizes $F^{(1)}$ subject to $F^{(1)} \leq K_i \; for \; i = 2,\ldots,q$, where K_i is a given upper bound on the value of $F^{(i)}$.
- Problem Variation 3 (PV_3): Identify a single Pareto-optimal solution (also known as a non-dominated or efficient solution) for each Pareto-optimal point, where a solution π is called Pareto-optimal with respect to $F^{(1)},\ldots,F^{(q)}$ if there is no other solution π', such that $F^{(i)}(\pi') \leq F^{(i)}(\pi)$ for $i = 1,\ldots,q$, with at least one of these inequalities being strict. The corresponding Pareto-optimal point is given by $\left(F^{(1)}(\pi),\ldots,F^{(q)}(\pi)\right)$.

We use the standard three-field notation $\alpha|\beta|\gamma$ introduced in Graham et al. (1979) to describe our scheduling problems. The α field describes the machine environment. If $\alpha = 1$, it implies that the scheduling is done on a single-machine. The β field defines the job-processing characteristics and constraints. When considering a multi-scenario scheduling problem we include the set of scenario-dependent parameters in this field. If processing times are scenario-dependent, we include $p_j^{(i)}$ in this field. We also include the rej entry in the β field for cases where rejection is allowed. The scheduling criteria appear in the γ field.

2. Brief literature review

The single-scenario variant of the makespan minimization problem with rejection, i.e., the $1|rej|C_{max}(\boldsymbol{A}) + RC$ problem, was studied by De et al. (1990). They observed that the objective function can be reformulated as

$$C_{max}(\boldsymbol{A}) + RC = \sum_{J_j\in \boldsymbol{A}} p_j + \sum_{J_j\in \widehat{\boldsymbol{A}}} e_j.$$

Therefore, if job J_j is included in $\boldsymbol{A}$, it contributes p_j to the objective function value, and if job J_j is included in $\widehat{\boldsymbol{A}}$, it contributes e_j to the objective function value. Accordingly, they concluded that the following lemma holds:

Lemma 1: The $1|rej|C_{max}(\boldsymbol{A}) + RC$ problem is optimally solvable in $O(n)$ time by applying the following rule for $j = 1,\ldots,n$: If $p_j \leq e_j$ then assign job J_j to set $\boldsymbol{A}$. Otherwise, assign J_j to set $\widehat{\boldsymbol{A}}$.

It follows from the above lemma that the optimal objective value of the $1|rej|C_{max}(\boldsymbol{A}) + RC$ problem is in fact $\sum_{j=1}^{n} min\{p_j, e_j\}$.

To the best of our knowledge, only Choi and Chung (2016) studied a multi-scenario scheduling problem with rejection to minimize the makespan. They studied the $1\left|rej, p_j^{(i)}\right| max_{i\in\{1,\ldots,q\}}\left\{G^{(i)} - G_{opt}^{(i)}\right\}$ problem, where $G_{opt}^{(i)}$ is the optimal (minimal) solution value under scenario i $(i = 1,\ldots,q)$ and

$$G^{(i)} = \sum_{J_j\in \boldsymbol{A}} p_j^{(i)} + \sum_{J_j\in \widehat{\boldsymbol{A}}} e_j.$$

They proved that (i) the problem is ordinary NP-hard even when $q = 2$; (ii) that when q is constant then the problem reduces to the min-max Shortest Path problem with q scenarios and thus admits an FPTAS (Fully Polynomial Approximation Scheme); (iii) that if q is arbitrary then the problem becomes strongly NP-hard; and that (iv) the special case where $p_j^{(i)} = p_j + \alpha^{(i)}$ ($\alpha^{(i)}$ is a scenario-dependent constant that is common to all jobs) is solvable in $O(n \log n)$ time. Moreover, they designed a 2-approximation algorithm for the general problem (with arbitrary q) which is based on LP relaxation.

3. Our results

Consider first PV_1, i.e., consider the $1\left|rej, p_j^{(i)}\right|\sum_{i=1}^{q}\theta^{(i)}\left(C_{max}^{(i)}(\boldsymbol{A}) + RC\right)$ problem. The fact that

$$\sum_{i=1}^{q}\theta^{(i)}\left(C_{max}^{(i)}(\boldsymbol{A}) + RC\right) = \sum_{i=1}^{q}\theta^{(i)}\left(\sum_{J_j\in \boldsymbol{A}} p_j^{(i)} + \sum_{J_j\in \widehat{\boldsymbol{A}}} e_j\right) = \sum_{J_j\in \boldsymbol{A}}\sum_{i=1}^{q}\theta^{(i)} p_j^{(i)} + \sum_{J_j\in \widehat{\boldsymbol{A}}} e_j \sum_{i=1}^{q}\theta^{(i)} = \sum_{J_j\in \boldsymbol{A}} p_j + \sum_{J_j\in \widehat{\boldsymbol{A}}} e_j,$$

where $p_j = \sum_{i=1}^{q}\theta^{(i)} p_j^{(i)}$ for $j = 1,\ldots,n$, implies that the following lemma holds:

Lemma 2: Any instance of the $1\left|rej, p_j^{(i)}\right|\sum_{i=1}^{q}\theta^{(i)}\,(C_{max}^{(i)}(\boldsymbol{A})+RC)$ problem reduces, in $O(nq)$ time, to an equivalent instance of the $1|rej|C_{max}(\boldsymbol{A})+RC$ problem by setting $p_j = \sum_{i=1}^{q}\theta^{(i)}\,p_j^{(i)}$ for $j=1,\dots,n$.

The following corollary is now straightforward from the results in Lemmas 1 and 2:

Corollary 1: The $1\left|rej, p_j^{(i)}\right|\sum_{i=1}^{q}\theta^{(i)}\,(C_{max}^{(i)}(\boldsymbol{A})+RC)$ problem is solvable in $O(nq)$ time by applying the following rule for $j=1,\dots,n$: If $p_j = \sum_{i=1}^{q}\theta^{(i)}\,p_j^{(i)} \le e_j$, then assign job J_j to set $\boldsymbol{A}$. Otherwise, assign J_j to set $\widehat{\boldsymbol{A}}$.

We then consider PV_2. We show that PV_2 is equivalent to the Multi-dimensional 0-1 Knapsack problem, where the problem parameters may take both negative and positive values. The fact that the less general Multi-dimensional 0-1 Knapsack problem, with non-negative value of parameters, is ordinary NP-hard for any constant number of dimensions, and is strongly NP-hard when *q* is arbitrary (see, Garey and Johnson, 1979) leads to the following theorem:

Theorem 1: PV_2 and PV_3 are at least ordinary NP-hard for any constant value of *q* and are strongly NP-hard when *q* is arbitrary.

Although there is a pseudo-polynomial time algorithm for special cases of the Multi-dimensional 0-1 Knapsack problem with both positive and negative parameters, when the number of dimensions is constant (e.g., for Multi-dimensional 0-1 Knapsack problem with only non-negative parameters, and for Subset Sum problem with both positive and negative parameters), we did not find an evidence for the existence of such an algorithm for our equivalent problem. Therefore, we still had to tackle the question whether PV_2 and PV_3 are strongly or ordinary NP-hard when *q* is constant.

We answer this question by showing that PV_2 and PV_3 are ordinary NP-hard for any constant value of *q*. We obtain this result by reducing each of the problems (PV_2 and PV_3) to a Multi-Criteria Shortest Path problem, which is solvable in pseudo polynomial time (Hassin 1992, Garroppo et al. 2010).. Therefore, the following theorem holds:

Theorem 2: PV_2 and PV_3 are ordinary NP-hard for any constant value of *q*.

We then show that our complexity results in Theorems 1 and 2 for PV_2 and PV_3 are also applicable for the absolute robustness problem variation (that is, the problem of finding a solution that minimizes the maximal objective value under all possible scenarios, i.e., that minimizes $\max_{i\in\{1,\dots,q\}}\{F^{(i)}\}$), leading for the following result as well:

Theorem 3: The absolute robustness problem variation is ordinary NP-hard for any constant value of *q* and is strongly NP-hard when *q* is arbitrary.

Finally, we consider a special case of PV_3, where for each job $J_j \in \boldsymbol{J}$ there are only two scenarios of processing times. We provide a fast O(*n*log*n*) time algorithm for finding the set of all supported solutions (where a solution is called supported if there exists a set of non-negative $\theta^{(i)}$ parameters ($i = 1,2,\dots,q$), such that this solution is optimal for PV_1). We note that the set of all supported solutions is a subset of the Pareto-optimal set of solutions.

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