Planning problem in Healthcare domain*

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1 Introduction

The complexity of planning in healthcare domain is an issue that is increasingly being highlighted by hospitals. Many healthcare problems belong to the family of Resource Constrained Project Scheduling Problems (RCPSP) that are NP-Hard (Garey M.R. and Johnson D.S. 1979) (Baptiste P. *et al.* 2006). The RCPSP problem consists in finding the best assignment of resources and start times to a set of activities. Scheduling problems have been the subject of many studies for decades in various fields (Anthony R.N. 1965) (Blazewicz J. *et al.* 2019), and they are of increasing interest in healthcare domain (Shnits B. *et al.* 2019). Through better patient care and better management of staff time schedules, health facilities want to reduce their costs while improving patient care. There is a rich literature on the variety and the description of these problems (Hall R.W. *et al.* 2012). Nowadays, schedules are mostly designed by hand, a difficult and time-consuming task that can be challenged by kinds of unexpected events. The structure of the problems that might be encountered differs according to the institutions, their size and the number of resources taken into account. The institutions' needs are various, and the criteria for evaluating a schedule may also change from one institution to another or from one department to another within the same institution. In this paper we present a 0-1 linear programming model able to cope with various real-world healthcare scenarios.

The rest of this article is structured as follows. In section 2, we describe and formalize our scheduling problem. In section 3 we present some instances and the corresponding results obtained by the CPLEX solver. In section 4, we conclude with some remarks and perspectives.

2 0-1 Linear programming model

The horizon H is decomposed into timeslots. We have a finite set of resources R. Each resource $r \in R$ is characterised by a set of properties Π_r that determines which roles a resource will be able to hold in an appointment. To each resource $r \in R$ is also associated a set of timeslot t such that $Available_t^r = 1$ if resource r is available at timeslot t. For example, an orthopedic surgeon who is available the first hour over $H =< t_1, t_2, t_3, t_4 >$ will be represented as ressource r with properties *orthopedic surgeon* and *orthopaedist* and with the set of available timeslots $< 1, 1, 0, 0 >$. He will be able to perform surgical operations and medical consultations. A is a set of appointments, such that each appointment $a \in A$ is characterized by its duration $duration_a$, a feasibility interval ES_a and LS_a, $qtreq_a^\pi$ the amount of resources with property π required by a. $Essential_a$ and $Emergency_a$ are two coefficients used to respectivly quantify the importance and the urgency of appointment

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a. They both occur as penalties in the objective function. Each appointment a is also defined by a set of resources R_a having one of the properties required by a and Π_a the set of properties required by a. $PreAssigned_a$ is a set of couples $(resource, property)$ pre-assigned to a. A_r is a set of appointments in which a resource r can participate.

We define decision variables x and y, with $x_a^{r,\pi} = 1$ if resource r with the property π is assigned to appointment a and $y_a^t = 1$ if appointment a starts at the timeslot t. Now, we are going to present the hard constraints mentioned above.

A resource may have multiple properties, and thus be able to perform multiple roles. However, resource r can only be allocated to appointment a with exactly one of its properties π if a is scheduled:

$$\forall\, a \in A, \forall\, r \in R_a, \sum_{\pi \in \Pi_r} x_a^{r,\pi} \leq 1 \tag{1}$$

If resource r does not have property π, it cannot be allocated to appointment a with this property π:

$$\forall\, r \in R, \sum_{\pi \in \Pi \setminus \Pi_r} \sum_{a} x_a^{r,\pi} = 0 \tag{2}$$

If resource r does not have any of the properties π required by appointment a, it cannot be allocated to a:

$$\forall\, a \in A, \sum_{r \in R \setminus R_a} \sum_{\pi \in \Pi} x_a^{r,\pi} = 0 \tag{3}$$

Each appointment a must be planned into a feasibilty interval determined by ES_a and LS_a:

$$\forall a \in A,\ ES_a \times \sum_{t \in H} y_a^t \leq t \times \sum_{t \in H} y_a^t \leq LS_a \tag{4}$$

If appointment a is planned, it is necessary to allocate the required quantity of resources with property π:

$$\forall\, a \in A, \forall\, \pi \in \Pi_a, \sum_{r \in R_a} x_a^{r,\pi} = qtreq_a^{\pi} \times \sum_{t \in [ES_a; LS_a]} y_a^t \tag{5}$$

Each resource r, allocated to appointment a, must be available for the complete duration of a:

$$\begin{aligned} &\forall\, a \in A, \forall\, r \in R_a, \forall\, t \in [ES_a; LS_a], \\ &\qquad duration_a \times \sum_{\pi \in \Pi_a} x_a^{r,\pi} - y_a^t \times \sum_{t'=t}^{t+duration_a-1} dispo_t^r \leq (1 - y_a^t) \times H \end{aligned} \tag{6}$$

An appointment a is at most scheduled once:

$$\forall\, a, \in A \sum_{t \in [ES_a; LS_a]} y_a^t \leq 1 \tag{7}$$

Resources are not allocated if the appointment a is not scheduled:

$$\forall\, a \in A, |R| \times |\Pi| \times \sum_{t \in [ES_a; LS_a]} y_a^t \geq \sum_{r \in R_a} \sum_{\pi \in \Pi_a} x_a^{r,\pi} \tag{8}$$

Resource r cannot be allocated to different appointments on same timeslot t:

$$\forall\, r \in R, \forall\, a \in A_r, \forall\, b \in A_r - \{a\}, \forall\, t \in [\max(ES_a; ES_b); \min(LS_a; LS_b)],$$

$$\sum_{\pi \in \Pi_r} x_a^{r,\pi} + \sum_{t'=t-duration_b+1}^{t} y_a^{t'} + \sum_{\pi \in \Pi_r} x_b^{r,\pi} + \sum_{t'=t-duration_b+1}^{t} y_b^{t'} \leq 3 \tag{9}$$

In most cases, an appointment is associated to a specific resource. The following constraint ensures that the resources r with their property π in $PreAssigned_a$ are allocated to appointment a:

$$\forall a \in A, \forall (r, \pi) \in PreAssigned_a,\; x_a^{r,\pi} = \sum_{t \in [ES_a; LS_a]} y_a^t \tag{10}$$

The quality of a solution is evaluated by an objective function f that computes the sum of the unplanned appointments $a \in A$, weighted by the importance factor $Essential_a$ and the sum of the differences between the start date of an appointment a and ES_a, weighted by the emergency factor $Emergency_a$. The purpose is to find a valid solution while minimizing the objective function defined in equation 11.

$$f = \sum_{a \in A} (1 - \sum_{t \in [ES_a; LS_a]} y_a^t) \times Essential_a + \sum_{a \in A} \sum_{t \in [ES_a; LS_a]} y_a^t \times \frac{t - ES_a}{LS_a - ES_a} \times Emergency_a \tag{11}$$

3 Experimentations and results

We generated instances from four different scenarios with the help of various planners from different health care facilities in France who face daily concrete problems.

Table 1. Description of the scenarios.

Scenario	Number of resources		Resources av.	Appointments dur.	Number of appointments		Horizon
					Per patient	Total	
SurgDep	π_1 = patient	16	100%	3 - 7 timeslots	1	16	23
	π_2 = surgeon	4	83%				
	π_3 = room	4	83%				
Admission	π_1 = patient	8	75%	1 - 2 timeslots	9	72	120
	π_2 = specialist	4	75%				
RehabCenter	π_1 = patient	24	100%	4 timeslots	4	96	20
	π_2 = doctor	12	100%				
	π_3 = physiotherapist	6	100%				
	π_4 = ergotherapist	6	100%				
CardioRehab	π_1 = patient	16	77%	2 timeslots	8	128	104
	π_2 = specialist	10	77%				

The characteristics of the scenarios are described in Table 1. For each scenario, we give the number of resources per property $\pi \in \Pi$, the rate of resources availability, the appointments duration, the number of appointments and the number of timeslots making up the horizon.

From each of these four scenarios, we generated three instances, starting with neither important nor urgent appointments and then increasing the number of essential (Ess) and urgent appointments (Em). We implemented the model under CPLEX and we ran tests on an Intel i5-8350U processor. We limited the computation time to two hours, and we reported the results in Table 2.

Table 2. Results of CPLEX

Instance name	Time	Objective value
$SurgDep\_Ess_0Em_0$	797	0
$SurgDep\_Ess_1Em_1$	2737	5.8160
$SurgDep\_Ess_2Em_2$	2085	19.4235
$CardioRehab\_Ess_0Em_0$	>7200	0-4
$CardioRehab\_Ess_1Em_1$	>7200	12.7059-16.7059
$CardioRehab\_Ess_2Em_2$	>7200	18.8015-23.0589
$Admission\_Ess_0Em_0$	246	0
$Admission\_Ess_1Em_1$	659	4.5098
$Admission\_Ess_2Em_2$	671	9.1734
$RehabCenter\_Ess_0Em_0$	1117	0
$RehabCenter\_Ess_1Em_1$	3015	14.8852
$RehabCenter\_Ess_2Em_2$	6956	23.7496

The objective value column corresponds to the objective function in equation 11. If CPLEX was unable to find an optimal solution within the time limit, we reported the upper and lower bound. The time column is the running time in seconds that CPLEX needs to work out the optimal solution. For all scenarios other than the $CardioRehab$ scenario, CPLEX found an optimal solution before the computation time limit. We noticed that increasing the number of important and urgent appointments implied an increase in computing time, except for the scenario $SurgDep$. The calculation time also increases with the number of appointments involved in the scenario.

4 Conclusion & perspectives

In this paper we have described and formalized concrete scheduling problems. We proposed a 0-1 linear programming model able to solve various scenarios in healthcare. We implemented this model under CPLEX and generated some instances in order to test it. The optimality has been reached for most instances and this model has proven to be effective on various scheduling issues in the medical domain. This will be a reliable benchmark to compare different approaches addressing these problems. We plan to develop a genetic algorithm to solve larger instances.

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