Towards the Optimisation of the Dynamic and Stochastic Resource-Constrained Multi-Project Scheduling Problem

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1 Introduction

Project scheduling is a rich and widely studied research area, most of the literature focuses on deterministic problems such as the *resource-constrained project scheduling problem* (RCPSP) and the *resource-constrained multi-project scheduling problem* (RCMPSP). The goals of RCPSP and RCMPSP typically minimise the total or average project completion times, which can be overly simplistic as it ignores the fact that projects may have different importance or rewards and may have deadlines with associated tardiness penalties. Solutions for such problems are deterministic schedules that show planned starting times of tasks.

The project execution frequently gets affected by many uncertainties; and project completion times deviate from the planned schedule. One uncertain element of project scheduling is task durations. Another uncertain element is stochastic resource availability. Resource availability could be affected by maintenance, breakdowns, personnel days off. In the literature, the stochastic equivalents of RCPSP and RMCPSP are called the *stochastic* RCPSP and the *stochastic* RCMPSP respectively. Most of the research in this field concerns only stochastic task durations e.g. Bruni, Pugliese, Beraldi & Guerriero. (2018). Only a few researched stochastic resource availability e.g. Wang, Chen, Mao, Chen & Li (2015).

Companies usually work on multiple projects at the same time to use their resources more effectively. On the arrival of a new project it should be added to execution as soon as possible without waiting for the completion of the previous schedule. Thus the arrival of a new project disrupts the previous schedules. The RCMPSP with random project arrivals is called *dynamic* RCMPSP (DRCMPSP). In the literature, the DRCMPSP considers the random arrival of new projects assuming all other project elements are deterministic.

Only a limited number of research considered the both dynamic project arrivals and stochastic task durations together e.g. Satic, Jacko & Kirkbride (2020). This problem is called the *dynamic and stochastic* RCMPSP. The dynamic and stochastic RCMPSP aims to find optimal schedules or scheduling policies that maximise the expected total discounted or time-average project reward minus the costs. In this paper we will focus on the former. We consider the dynamic and stochastic RCMPSP with project arrivals and stochastic task durations. We model the problem as an infinite-horizon discrete-time *Markov decision process* (MDP).

2 Modelling Framework

In this study, we assume the dynamic and stochastic RCMPSP contains J project types where the project type, j, determines the characteristics such as a new project arrival

Table 1. Example decision state for a problem with two project types, each with three tasks.

	Tasks	Remaining due date
Project type 1 :	$x_{1,1}$ $x_{1,2}$ $x_{1,3}$	d_1
Project type 2 :	$x_{2,1}$ $x_{2,2}$ $x_{2,3}$	d_2

probability (λ_j), number of tasks (I_j), project network, resource requirement per unit time of a task i ($b_{j,i}$), task completion probabilities ($\gamma_{j,i}$), minimal possible completion time ($t_{j,i}^{min}$), maximal possible completion time ($t_{j,i}^{max}$), project due date (F_j), reward (r_j) and tardiness cost (w_j). We model this problem as an infinite horizon *Discrete Time Markov Decision Process* (DT-MDP) which is defined by five elements: time horizon, decision state space, action set, transition function and profit function.

In a DT-MDP, the decision maker takes an action a for a decision state s at a decision epoch that occur at fixed intervals. The period between two consecutive decision epochs is a single unit of time, called a period.

The system information at a decision epoch is called a decision state (s). An example decision state for a two project-type problem is given in Table 1, where $x_{j,i}$ is the remaining task processing time to the latest possible task completion time (t_j, i^{max}) and d_j is the remaining time until the due date. For tasks awaiting processing we set $x_{j,i}$ to -1.

The action a represents the processing decision of pending tasks ($x_{j,i} = -1$) of the decision state (s). An action must satisfy two conditions: there must be enough resources (B) available to begin processing these new tasks ($\sum_{j=1}^{J}\sum_{i=1}^{I_j} b_{j,i}(\mathcal{I}\{a_{j,i}=1\} + \mathcal{I}\{x_{j,i} > 0\}) \leq B$) and all predecessor tasks ($\mathcal{M}_{j,i}$) of task i must be completed ($\sum_{m\in\mathcal{M}_{j,i}} x_{j,m} = 0$).

After the selected action a is applied in the decision state s, the system transforms from one state to another (s') at the next decision epoch according to a transition function $P(s'|s,a)$.

$$P(s'|s,a) = \prod_{j=1}^{J}\prod_{i=1}^{I_j} P(x'_{j,i}|x_{j,i}+a_{j,i}) \tag{1}$$

$$P(x'_{j,i}|x_{j,i}+a_{j,i}) = \begin{cases} \lambda_j\gamma_{j,i}(x_{j,i}+a_{j,i}), & \text{for } 1 \leq x_{j,i}+a_{j,i} \leq 1+t_{j,i}^{max}-t_{j,i}^{min}, \\ & x'_{j,i} = -1, i = I_j \\ (1-\lambda_j)\gamma_{j,i}(x_{j,i}+a_{j,i}), & \text{for } 1 \leq x_{j,i}+a_{j,i} \leq 1+t_{j,i}^{max}-t_{j,i}^{min}, \\ & x'_{j,i} = 0, i = I_j \\ \gamma_{j,i}(x_{j,i}+a_{j,i}), & \text{for } 1 \leq x_{j,i}+a_{j,i} \leq 1+t_{j,i}^{max}-t_{j,i}^{min}, \\ & x'_{j,i} = 0, i < I \\ 1-\gamma_{j,i}(x_{j,i}+a_{j,i}), & \text{for } 1 \leq x_{j,i}+a_{j,i} \leq 1+t_{j,i}^{max}-t_{j,i}^{min}, \\ & x'_{j,i} = x_{j,i}+a_{j,i}-1 \\ \lambda_j, & \text{for } x_{j,i}+a_{j,i} = 0, x'_{j,i} = -1, i = I_j \\ 1-\lambda_j, & \text{for } x_{j,i}+a_{j,i} = 0, x'_{j,i} = 0, i = I_j \\ 1, & \text{for } x_{j,i}+a_{j,i} = 0, x'_{j,i} = 0, i < I_j \\ 1, & \text{for } x_{j,i}+a_{j,i} > 1+t_{j,i}^{max}-t_{j,i}^{min}, \\ & x'_{j,i} = x_{j,i}+a_{j,i}-1 \\ 1, & \text{for } x_{j,i}+a_{j,i} = -1, x'_{j,i} = -1 \end{cases} \tag{2}$$

The profit function ($R_{s,a}$) is the sum of rewards (r_j) of completed projects in the current period minus the tardiness cost of any late completions.

$$\begin{aligned} R_{s,a} = &\sum_{j=1}^{J} r_j \mathbb{E}\Big[\mathcal{I}\big\{\big(x_{j,I} \geq 1 \vee (x_{j,I} = -1 \wedge a_{j,I} = 1)\big) \wedge x'_{j,I} \leq 0\big\}\Big] \\ &- \sum_{j=1}^{J} w_j \mathbb{E}\Big[\mathcal{I}\big\{\big(x_{j,I} \geq 1 \vee (x_{j,I} = -1 \wedge a_{j,I} = 1)\big) \wedge x'_{j,I} \leq 0 \wedge d_j = 0\big\}\Big]. \end{aligned} \tag{3}$$

3 Solution Methods

We compare six different solution approaches which are: a dynamic programming algorithm (DP), an approximate dynamic programming algorithm (ADP), an optimal reactive baseline algorithm (ORBA), a genetic algorithm (GA), a rule-based algorithm (RBA) and a worst decision algorithm (WDP).

The dynamic programming value iteration algorithm is used to determine an optimal policy that maximises the discounted long-time profit. We tested computational limitations of DP in the dynamic and stochastic RCMPSP.

ADP replaces the true value function of the Bellman's equation with an approximate one to overcome the curse of dimensionality problem of DP. We built a linear approximate value function with two state information as decision variables and weighted them using coefficients. The decision variables are the number of the period spent on processing each type of projects and the number of allocated resources between project types.

We used three reactive scheduling heuristics which are ORBA, GA and RBA. GA and RBA methods are popular for both dynamic and static RCMPSP problems thus we added them our comparison to evaluate their performance. Reactive scheduling methods do not consider the future uncertainties while generates schedules; then they fix these schedules at each distribution (Rostami, Creemers & Leus 2018).

Reactive scheduling methods generate a new baseline schedule and convert it to an action for each state. ORBA and GA seek to maximise the profit and uses the total completion time as tiebreakers between the schedules with equal reward. If several schedules have equal rewards and equal completion times, the models prioritise smallest numbered project type. We used a population of 100 with 100 generations in GA. RBA uses the longest processing time first rule to schedule. If several tasks have equal duration, RBA selects one at random.

WDP, using a dynamic programming value iteration algorithm, seeks a non-idling policy to minimise the average profit per unit time. We used this method in our comparison to show the profit of the worst non-idling policy.

4 Algorithm evaluation

All tests are performed on a desktop computer with Intel i5-6500T CPU with 2.50 GHz clock speed and 32 GB of RAM. JuliaPro 1.3.1.2 is used for coding the model, solution approaches and problems. Three dynamic and stochastic RCMPSPs are generated and tested consecutively from 1% to 90% project arrival probabilities, incremented by 10%.

Table 2 shows the discounted long-term profits of six algorithms with 95% discount rate. The policies of reactive scheduling algorithms are closer to optimum with low project arrival rates such as 1% where system is closer to static and they diverge from the optimum as arrival probability increases. ADP suffers at low arrival probabilities but produces equal or better results to ORBA after %30 arrival probability.

Table 2. Discounted long-term profits

λ_j	1%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Two projects and two tasks problem										
DP	6.16	16.23	22.55	26.32	28.76	30.46	31.70	32.64	33.38	33.96
ADP	3.42	13.45	18.02	22.04	26.57	27.91	28.86	29.54	30.05	30.44
ORBA	5.99	15.64	21.31	24.54	26.55	27.89	28.83	29.51	30.02	30.41
GA	5.74	14.70	18.54	20.73	19.64	24.97	20.60	22.74	21.62	20.85
RBA	5.49	13.45	16.36	16.99	16.84	16.44	16.00	15.59	15.26	15.04
WDA	5.38	12.96	15.37	15.73	15.50	15.16	14.78	14.26	13.68	12.96
Two projects and three tasks problem										
DP	10.17	18.79	23.08	25.43	26.91	27.92	28.65	29.20	29.63	29.98
ADP	6.85	18.51	22.70	25.12	26.53	27.47	28.14	28.64	29.03	28.89
ORBA	10.15	18.68	22.86	25.12	26.53	27.47	28.14	28.64	29.03	29.34
GA	10.15	18.68	22.86	25.12	26.53	27.47	28.14	28.64	29.03	29.34
RBA	10.11	18.42	22.37	24.44	25.69	26.50	27.05	27.45	27.75	27.99
WDA	9.79	17.13	20.07	21.44	22.20	22.66	22.95	23.15	23.29	23.40
Three projects and two tasks problem										
DP	15.14	28.36	32.63	34.92	36.98	38.74	40.19	41.37	42.33	43.12
ADP	10.50	25.48	29.15	31.83	34.23	36.28	37.99	39.39	41.23	42.03
ORBA	14.88	27.48	30.73	31.75	32.31	32.73	33.10	33.43	33.74	34.03
GA	14.63	26.79	29.37	29.84	29.88	29.97	30.65	30.60	31.29	31.62
RBA	14.61	25.96	28.64	29.96	31.21	32.48	33.73	34.93	36.06	37.11
WDA	13.82	22.59	23.31	23.26	23.14	23.05	23.04	23.10	23.23	23.40

5 Conclusion

We consider the RCMPSP with uncertain project arrivals and stochastic task durations as an infinite-horizon DT-MDP. We used six approaches and compared their results. We also tested the computational limits of the DP on the dynamic and stochastic RCMPSPs. We observed that DP suffers from the curse of dimensionality even for the small size problems and results of reactive scheduling methods deteriorate compared to optimum results as stochasticity increases. ADP performs similar or better than ORBA, which is the second best method, after 30% arrival probability. More detailed description of the model and more extensive results can be found at Satic et al. (2020).

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