A New Lower Bound Approach for the Multi-mode Resource Constrained Project Scheduling Problem

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Keywords: MRCPSP, project scheduling, lower bounds.

1 Introduction and problem description

The multi-mode resource constrained project scheduling problem (MRCPSP) is an extension of the resource-constrained project scheduling problem (RCPSP). Besides the decision of starting time, a mode has to be chosen for each activity.

The objective of the MRCPSP is to find the minimum feasible makespan of the project. The project consists of a set of activities $A = \{0, ..., n+1\}$. Each activity i can be executed in different modes. Therefore, for each activity i a set of modes M_i is given. Mode m has a duration $d_{i,m} \in \mathbb{Z}^+$ as well as a resource consumption $r_{i,m,k} \in \mathbb{Z}^+$ for each resource $k \in \mathcal{R} \cup \mathcal{R}^n$. The duration and the resource consumption of an activity varies with respect to the chosen mode. On the one hand, a set of renewable resources $\mathcal{R}$ is given which are available per time unit. On the other hand, a set of non-renewable resources $\mathcal{R}^n$ exists which are available through out the whole project. Activities have precedence constraints between each other. Set $E = \{(i,j) : i, j \in A\}$ indicates the precedence relations. Every activity has to be set to a mode and a starting time, while respecting all precedence and resource constraints. A mathematical model for the MRCPSP was first described by Talbot (1982):

$$\min \sum_{ES_{n+1}}^{LS_{n+1}} x_{n+1,1,t} \cdot t \tag{1}$$

$$\text{s.t.} \sum_{m \in M_i} \sum_{t=ES_i}^{LS_i} x_{i,m,t} = 1 \qquad \forall i \in A \tag{2}$$

$$\sum_{m \in M_i} \sum_{t=ES_i}^{LS_i} x_{i,m,t} \cdot (t + d_{i,m}) \leq \sum_{m \in M_j} \sum_{t=ES_j}^{LS_j} x_{j,m,t} \cdot t \qquad \forall (i,j) \in E \tag{3}$$

$$\sum_{i \in A} \sum_{m \in M_i} \sum_{t=ES_i}^{LS_i} x_{i,m,t} \cdot r_{i,m,k} \leq R_k \qquad \forall k \in \mathcal{R}^n \tag{4}$$

$$\sum_{i \in A} \sum_{m \in M_i} \sum_{q=\max(ES_i, t-d_{i,m}+1)}^{\min(t, LS_i)} x_{i,m,q} \cdot r_{i,m,k} \leq R_k \qquad \forall k \in \mathcal{R}, \forall t \in T \tag{5}$$

$$x_{i,m,t} \in \{0,1\} \qquad \forall i \in A, \forall m \in M_i, t = ES_i, \ldots, LS_i \tag{6}$$

The binary decision variables $x_{i,m,t}$ are set to one if and only if the activity i is executed in mode m and if it starts at time period t. We consider the non-preemptive case of the MRCPSP, which means that an activity cannot be interrupted or change its mode, once it is started. Note that in the model above the index t denotes the starting period and not the completion period as introduced by Talbot.

The objective function (1) minimizes the starting time of the dummy activity $n+1$. As this denotes the end of the project, the makespan of the project is minimized. Term (2) ensures that each activity is assigned to exactly one mode and to one starting time. Constraints (3) guarantee the precedence relations, i.e., an activity i must be finished before activity j can start if $(i,j) \in E$. Inequalities (4) and (5) restrict the resource consumptions for the non-renewable and renewable resources, respectively. The binary decision variables are defined in (6).

The MRCPSP is a $\mathcal{NP}$-hard problem, even finding a feasible mode assignment is $\mathcal{NP}$-complete if the instance has more than one non-renewable resource (Kolisch and Drexl (1997)). Lower bounds can be used either to evaluate the quality of a solution or as bounds in an exact approach to reduce the solution space. Therefore this work presents new lower bound approaches for the MRCPSP.

2 A new lower bound approach

Several lower bound procedures for the MRCPSP already exist in the literature. The most common one is the Critical Path Lower Bound (CP-LB) which only considers the precedence constraints and relaxes all resource constraints (Kelley (1963)). Since every activity has more than one mode the mode with the shortest duration is always chosen for the CP-LB.

More complex lower bounds were presented as well. Maniezzo and Mingozzi (1999) presents several LP relaxations. Pesch (1999) uses an adaptation of the Talbot (1982) algorithm for generating lower bounds. The approaches of Zhu *et al.* (1997) and Stürck and Gerhards (2018) are based on calculating new earliest starting times for the activities which lead to new lower bounds. But there are a lot more lower bound procedures for the RCPSP. For an example the work of Klein and Scholl (1999) alone presents 17 different approaches for lower bounds for the RCPSP.

This work will present a new lower bound approach for the MRCPS. It is based on the *Capacity Bound* for the RCPSP in Klein and Scholl (1999). Klein and Scholl (1999) described the *Capacity Bound* as follows: for each activity the duration is multiplied with the renewable resource consumption of the activity. These products are summed up and divided by the available resource per time period:

$$\mathit{Capacity\ Bound} = \max\{\lceil \sum_{i \in A} r^r_{i,k} \cdot d_i / R_k \rceil : k \in \mathcal{R}^r\} . \tag{7}$$

This operation is done for every renewable resource. The maximal rounded up quotient determines the lower bound for the RCPSP.

To use this approach for the MRCPSP it has to be adapted. We combine the *Capacity Bound* with a feasible mode assignment. Therefore we call this bound the *Feasible Mode Capacity Bound*. The used feasible mode assignment is inspired by the MIP approach of Toffolo *et al.* (2016). The problem of the mode selection is redefined to a multidimensional knapsack problem:

$$\min \sum_{i \in A} \sum_{m \in M_i} y_{i,m} \cdot d_{i,m} \tag{8}$$

$$\text{s.t.} \sum_{m \in M_i} y_{i,m} = 1 \qquad \forall i \in A \tag{9}$$

$$\sum_{i \in A} \sum_{m \in M_i} y_{i,m} \cdot r^n_{i,m,k} \leq R^n_k \qquad \forall k \in \mathcal{R}^n \tag{10}$$

$$y_{i,m} \in \{0,1\} \qquad \forall i \in A, \forall m \in M_i \tag{11}$$

The binary decision variables $y_{i,m}$ are set to one if and only if the activity i is executed in mode m. The constraint (9) assigns exactly one mode to each activity. Term (10) ensures that the non-renewable resources are not exceeded. The objective function (8) is just subsidiary for finding a feasible mode assignment.

These two approaches can be combined a new lower bound procedure: the *Feasible Mode Capacity Bound*:

$$\min B_l = \sum_{i \in A} \sum_{m \in M_i} y_{i,m} \cdot d_{i,m} \cdot r^r_{i,m,k} \tag{12}$$

$$\text{s.t.} \sum_{m \in M_i} y_{i,m} = 1 \qquad \forall i \in A \tag{13}$$

$$\sum_{i \in A} \sum_{m \in M_i} y_{i,m} \cdot r^n_{i,m,k} \leq R^n_k \qquad \forall k \in \mathcal{R}^n \tag{14}$$

$$y_{i,m} \in \{0,1\} \qquad \forall i \in A, \forall m \in M_i \tag{15}$$

The aim is to find a feasible mode assignment with the minimal renewable resource usage B_l for each renewable resource $l \in \mathcal{R}^r$. The binary decision variables (15) are set to one if and only if the activity i is executed in mode m. The objective function (12) is similar to Term (7) with the addition of $y_{i,m}$ which ensures that only the chosen modes are considered. With (13) each activity is assigned to exactly one mode. Constraint (14) considers all non-renewable resources $k \in \mathcal{R}^n$.

This mathematical model ((12) – (15)) is solved for each renewable resource $l \in \mathcal{R}^r$. In the next step the *Feasible Mode Capacity Bound* can be solved:

$$\textit{Feasible Mode Capacity Bound} = \max\{\lceil B_l/R_l \rceil : l \in \mathcal{R}^r\} . \tag{16}$$

Each quotient is rounded up since only integer periods are considered. The maximal quotient determines the *Feasible Mode Capacity Bound*. The next section will display the computational experiments.

3 Computational experiments

The experiments were done on the MMLIB instances presented by Van Peteghem and Vanhoucke (2014) and carried out on a PC with an Intel Xeon X5650 CPU at 2.66 GHz. The algorithm is implemented in C# and CPLEX 12.6.3 was used as solver.

Although finding a feasible mode assignment is already $\mathcal{NP}$-complete if the instance has more than one non-renewable resource (Kolisch and Drexl (1997)) the procedure is quite fast. The computation of the lower bounds took 24.03 seconds on average, with a minimum of 0.01 seconds for 204 instances and a maximum of 109.91 seconds for an MMLIB+ instance with 100 activities and 9 modes. Table 1 shows the results for the computational experiments.

Table 1. Computational experiments for the *Feasible Mode Capacity Bound* on the MMLIB

	MMLIB50	MMLIB100	MMLIB+	Sum
total number of instances	540	540	3 240	4 320
Best known solution = CP-LB	229	264	473	966
Feasible Mode Capacity Bound > CP-LB	107	120	1 878	2 105

For 966 instances the best known solution is already equal to the *Critical Path Bound* and therefore optimal. The presented procedure was able to find a better *Feasible Mode Capacity Bound* compared to *Critical Path Bound* for 2 105 of the remaining 3 354 instances.

4 Conclusion

This work presented a new approach for computing lower bounds for the MRCPSP. The computational experiments showed that the procedure is able to improve the lower bound for 67.76 % of the MMLIB instances without a known optimum. Furthermore, the experiments showed that the computation time of the approach is reasonably low with a few seconds for most of the MMLIB instances.

References

Kelley, J. E. 1963, “The critical-path method: Resources planning and scheduling", *Industrial Scheduling*, Vol. 13, no. 1, pp. 347-365.

Klein, R. and Scholl, A. 1999, “Computing lower bounds by destructive improvement: An application to resource-constrained project scheduling", *European Journal of Operational Research*, Vol. 112, no. 2, pp. 322-346.

Kolisch, R. and Drexl, A. 1997, “Local search for nonpreemptive multi-mode resource-constrained project scheduling", *IIE Transactions*, Vol. 29, no. 11, pp. 987-999.

Maniezzo, V. and Mingozzi, A. 1999, “A Heuristic Procedure For the Multi-mode Project Scheduling Problem Based on Benders Decomposition", In: *Project Scheduling: Recent Models, Algorithms and Applications* eds: Węglarzz, J., pp. 179-196, Kluwer(Boston).

Pesch, E. 1999, “Lower bounds in different problem classes of project schedules with resource constraints", In: *Project Scheduling: Recent Models, Algorithms and Applications* eds: Węglarzz, J., pp. 179-196, Kluwer(Boston).

Stürck, C. and Gerhards, P. 2018, “Providing Lower Bounds for the Multi-Mode Resource-Constrained Project Scheduling Problem", In: *Operations Research Proceedings 2016* eds: Fink, A., Fügenschuh, A. and Geiger, M. J., pp. 551-557, Springer (Cham).

Talbot, F. B. 1982, “Resource-constrained project scheduling with time-resource tradeoffs: The nonpreemptive case", *Management Science*, Vol. 28, no. 10, pp. 1197-1210.

Toffolo, T., Santos, H. G., Carvalho, M., and Soares, J. A., “An integer programming approach to the multimode resource-constrained multiproject scheduling problem", *Journal of Scheduling*, Vol. 19, no. 3, pp. 295-307.

Van Peteghem, V. and Vanhoucke, M. 2014, “An experimental investigation of metaheuristics for the multi-mode resource-constrained project scheduling problem on new dataset instances", *European Journal of Operational Research*, Vol. 235, no. 1, pp. 62-72.

Zhu, G., Bard, J. F. and Yu, G. 2006, “A branch-and-cut procedure for the multimode resource-constrained project-scheduling problem", *INFORMS Journal on Computing*, Vol. 18, no. 3, pp. 377-390.