A Stochastic Programming Model to Schedule Projects under Cash Flow Uncertainty

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1 Introduction

The uncertainties inherent in project scheduling lead to a challenging problem for project managers. Many studies in the relevant literature ignore this factor even though the consideration of uncertainty is critical. Several of the stochastic project scheduling studies consider only the uncertainty in activity durations. However, there exists other uncertainty-inducing factors such as disruptions in resource usages/availabilities or delays in cash flows that are important while managing projects (see Hazir and Ulusoy 2019 for an extensive review of the subject). The delays in cash inflows are particularly common, because the financial positions of the clients depend on several uncontrollable factors. To the best of our knowledge, there does not exist any study in the project scheduling literature that considers delays in client payments.

In this article, we investigate the uncertainties associated with delays in client payments and model the effects of these in the net present value (NPV), which is a common criterion used to assess the financial feasibility of the projects. We formulated our project scheduling problem as a two-stage stochastic mixed integer program. The activity start times are the main decision variables in the model. The actual client payment times are represented as second-stage decision variables. The objective function maximizes the expected NPV.

Our research is related to two main streams of past research on project scheduling with financial objectives. The articles that proposed approaches different from stochastic programming models are grouped within the first category. Our research belongs to the second category of articles in which the stochastic programming models are formulated.

Russell (1970) is known as the leading work in the first category of articles. They maximized the NPV in the objective function and proposed a first-order Taylor series approximation based approach to linearize it. This work was followed by many others that consider NPV in the objective function (Elmaghraby and Herroelen 1990, Herroelen and Gallens 1993, Kazaz and Sepil 1996). Buss and Rosenblatt (1997) assumed uncertainty in activity durations and determined the optimal amount of delays beyond the earliest activity start times. Wiesemann et al. (2010) considered uncertainty in activity durations and cash flow amounts based on a discrete set of scenarios. They enforced nonanticipativity by imposing target process times for activities at each scenario. They solved the model using a branch and bound algorithm. Sobel et al. (2009) also considered randomness in activity durations and cash flow amounts. They investigated the optimal adaptive schedule by developing a continuous-time Markov decision chain model. Creemers et al. (2015) used a stochastic dynamic programming approach in their study where they considered technological uncertainty and stochastic activity durations. Their model incorporates the risk of activity failure which may result in project failure. Creemers (2018) maximized the ENPV in the objective function by studying with stochastic activity durations that are modeled using phase-type distributions. They used a new continuous-time Markov chain and a backward stochastic dynamic program to determine the optimal policy.

In the second category of articles, Klerides and Hadjiconstantinou (2010) developed two path-based two-stage stochastic integer programming models. The models include uncertainty in activity durations and costs. The main decision is about the execution mode of an activity. The objective functions in the models minimize the total cost and expected project duration, respectively. They proposed a decomposition-based algorithm to solve the model. Davari and Demeulemeester (2019) considered uncertainty in a resource-constrained project scheduling problem (RCPSP). To deal with the uncertainty, they studied the chance-constrained resource-constrained project scheduling problem (CC-RCPSP), which was introduced recently. They formulated the sample average approximation (SAA) counterpart of the CC-RCPSP (SAA-RCPSP) due to the large size of finite supporting set of realizations. They used a branch-and-bound algorithm (B&B) to solve the SAA-RCPSP. Lamas and Demeulemeester (2015) also modeled an RCPSP. Their model contains stochastic activity durations. They aimed to create a new procedure for generating a baseline schedule for the problem. They also studied the SAA of their original model. They - implemented a branch-and-cut algorithm to find a robust baseline schedule considering a new robustness measure that they introduced.

Our study is different from both category of articles, because we propose a two-stage stochastic programming model that consider uncertainty in client payment delays.

2 Problem Description

We formulated our problem as a two-stage stochastic mixed integer program (SMIP). The start time for each activity is set at the first stage under uncertainty related to the delay of client payment times. The actual times of cash inflows, which depend on activity completion times and delays in payments, are modeled as the second-stage variables. The cash outflows are observed at the beginning of each activity, however the inflows are observed at the completion of a given set of activities. A deadline is enforced on the project completion time, but the client payments are allowed to be received after this deadline. The objective is to maximize the expected net present value of the project. Following is a detailed description of our formulation.

Indices and Sets:

i, j: node (i.e. project activity) index
(i, j): index of the arc from node i to node j
t: time index
ω: scenario index
V: set of all nodes
V^I: set of cash-generating activities
E: set of all arcs (i.e. immediate precedence relationships)
T^i: set of time periods at which activity $i \in V$ can start (i.e. time periods between the earliest and latest start times for an activity)
P^i: set of time periods at which payment can be received for the completion of activity $i \in V^I$ (i.e. time periods after the earliest completion time for an activity)
Ω: set of all scenarios

Parameters:

p_i: duration of activity $i \in V$
$\overline{d}$: deadline for the completion of the project

n: number of activities of the project, excluding dummy nodes for project beginning (node 0) and project completion (node $n+1$)
c_i^{F+}: cash inflow due to the completion of activity $i \in V^I$ ($c_i^{F+} > 0$)
c_i^{F-}: cash outflow due to the initiation of activity $i \in V$ ($c_i^{F-} < 0$)
β: discount rate per time period
e_i^{ω}: delay in payment after the completion of activity $i \in V^I$ under scenario $\omega \in \Omega$

First-Stage Decision Variables:

$$x_{it} = \begin{cases} 1 & \text{if activity } i \in V \text{ starts at time } t \in T^i; \\ 0 & \text{otherwise,} \end{cases}$$

Second-Stage Decision Variables:

$$q_{it}^{\omega} = \begin{cases} 1 & \text{if payment for activity } i \in V^I \text{ is received at time } t \in P^i \text{ under scenario } \omega \in \Omega; \\ 0 & \text{otherwise,} \end{cases}$$

$$\max \quad \sum_{i \in V} \sum_{t \in T^i} \frac{c_i^{F-}}{(1+\beta)^t} x_{it} + \mathcal{Q}(\mathbf{x}) \tag{1}$$

s.t.

$$\sum_{t \in T^i} x_{it} = 1 \qquad \forall i \in V \tag{2}$$

$$\sum_{t \in T^j} t x_{jt} \geq \sum_{t \in T^i} t x_{it} + p_i \qquad \forall (i,j) \in E \tag{3}$$

$$\sum_{t \in T^{n+1}} t x_{(n+1)t} + p_{n+1} \leq \overline{d} \tag{4}$$

$$x_{it} \in \{0,1\} \qquad \forall i \in V, t \in T^i \tag{5}$$

where $\mathcal{Q}(\mathbf{x}) = E_{\xi}[\mathrm{Q}(\mathbf{x}, \xi(\omega))]$ is the expected recourse function, and

$$\mathrm{Q}(\mathbf{x}, \xi(\omega)) = \max \sum_{i \in V^I} \sum_{t \in P^i} \frac{c_i^{F+}}{(1+\beta)^t} q_{it}^{\omega}$$

s.t.

$$\sum_{t \in P^i} t q_{it}^{\omega} = \sum_{t \in T^i} t x_{it} + p_i + e_i^{\omega} \qquad \forall i \in V^I \tag{6}$$

$$\sum_{t \in P^i} q_{it}^{\omega} = 1 \qquad \forall i \in V^I \tag{7}$$

$$q_{it}^{\omega} \in \{0,1\} \qquad \forall i \in V^I, t \in P^i \tag{8}$$

The objective function (1) includes the net present value of the summation of the cash outflows that depend on the activity start times, and the expected second-stage function. The expected second-stage function maximizes the net present value of the cash inflows that occur after a possible delay following the activity completion time. First-stage constraints are represented by (2)-(5). We assume that the earliest/latest start times of each activity were calculated in advance using the forward-backward passes. Constraints (2) ensure that an activity starts in between its earliest and latest start time. Constraints (3) maintain that the start time of an activity is greater than or equal to the completion time of the activity that immediately precedes it. Constraints (4) require that the project is completed before the deadline. Constraints (5) enforce binary restrictions on the first-stage variables.

In the second stage, constraints (6) calculate the time of cash inflow by considering possible delay after the activity completion time.Constraints (7) ensure that the cash inflow for a completed activity is received as a lump-sum amount at a single period. Constraints (8) represent the binary restrictions on the second-stage variables.

3 Conclusion

We apply a sample average approximation (SAA) algorithm to solve the SMIP model (Kleywegt et al. 2002). The SAA algorithm approximates the true objective value by solving instances created by sampling N scenarios. The algorithm can be used to assess the optimality gap as well as for obtaining a solution. The SAA solves M instances, each having N scenarios to obtain an estimate of the lower bound. Then, an upper bound is calculated for each instance solution by evaluating its objective value over N' scenarios. Note that N' is generally set to a much larger value than N.

In our experiments, we intend to illustrate the impact of randomness in the delay of client payments into the activity start times. We also show the benefit of considering uncertainty in payment delays. We examine how the deadline constraint (i.e. constraint (4)) affects the net present value and optimal activity start times.

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