A mixed integer programming approach for scheduling aircraft arrivals at terminal airspace fixes and runway threshold

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1 Introduction and motivation

The continuous growth of air transportation demand exposes the Air Traffic Management (ATM) system to a serious risk of saturation, especially the terminal airspace – known as the Terminal Maneuvering Area (TMA) – of hub airports. A better utilization of current airport infrastructures can alleviate the saturation problem, for instance by reducing delays.

Several research works in the literature are interested in the optimization of runway sequences. However, most of such work focuses on the runway system. For instance, Beasley *et al.* (2000) propose a Mixed Integer Programming (MIP) approach for scheduling aircraft landings on multiple runways. Furini *et al.* (2015) consider the problem of scheduling simultaneously aircraft take-offs and landings on a single runway. Recently, Prakash *et al.* (2018) adapt the model of Beasley *et al.* (2000) to incorporate take-offs, and attempt at minimizing the schedule makespan. Readers may refer to the survey of Bennell *et al.* (2011) for a comprehensive review of the proposed solution approaches in the literature.

In this work, we propose a new Mixed Integer Linear Programming (MILP) approach for sequencing and scheduling aircraft arrivals at critical TMA points, called the Initial Approach Fixes (IAF), as well as on the runway threshold. What drives this work is that runway operations depend on other operations from the TMA, while the majority of works in the literature consider the runway as an independent resource. Another motivation comes from the observation of inbound traffic in the TMA, which reveals unbalanced traffic flows among the IAFs. Our objective is to re-distribute arriving aircraft among the existing TMA fixes and runways, so as to balance traffic volume. Preliminary results show that the average delay at runways can be reduced using our model.

2 Analogies and complexity

The problem of scheduling aircraft arrivals considering only runways is called the Aircraft Landing Problem (ALP) in the literature. It is similar to a number of classical combinatorial optimization problems, namely, the job-shop scheduling problem, the Traveling Salesman Problem (TSP), and the Vehicle Routing Problem (VRP).

The analogy between the ALP and a job-shop scheduling problem can be derived from Beasley *et al.* (2000) as follows. Runways correspond to machines, aircraft to be sequenced correspond to jobs, and the *safety separation* between two successive landings (see *Section* 3) correspond to the sum of the processing time of the previous job, and the sequence-dependent set up time. The typical objective function is to minimize the landing of the last aircraft in the sequence. It corresponds to minimizing the makespan of the schedule. In this case, if we consider a single runway, the problem is then equivalent to the TSP.

The analogy between the ALP and the VRP is pointed out in Briskorn and Stolletz (2014). In this context, runways represent vehicles to dispatch, aircraft represent customers to serve, and the *safety separation* between two successive landings correspond to the distance between two customers. The target landing times usually considered in the ALP, correspond to times at which customers prefer to be served. Upper and lower bounds on these target times correspond to the classical time-window restrictions. The objective function consists then in landing (serving) each aircraft (customer) within its time window, such that the cost associated to deviations from target times is minimized.

It can be deduced from these analogies that even the simplest versions of the ALP, involving only runways, are NP-hard problems. However, some versions of the ALP that impose particular restrictions can be solved in polynomial times. These include ALP with Constrained Position Shifting (CPS), as shown in Balakrishnan and Chandran (2006), and the ALP with aircraft classes, presented in Briskorn and Stolletz (2014).

3 Problem context and formulation

In this work, we consider the problem of sequencing and scheduling aircraft arrivals at two levels: first, over initial approach fixes, which are the points where the initial approach segment of an Instrument Flight Rule (IFR) approach begins (Figure 1), and then at the runway threshold.

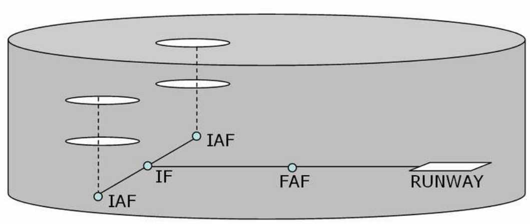

Fig. 1. A diagram of a typical IFR approach, with two IAFs and one runway (IF and FAF are intermediate way points on the approach trajectory). Source: Kelly and Painter (2006)

We propose a MILP formulation of this problem. The given data includes, for each aircraft: a target landing time, a latest acceptable landing time (based on fuel considerations), and the transition time between each IAF and the runways. Two types of safety constraints are considered:

- The pairwise separation of 3 Nautical Miles (NM) at each IAF.
- the Wake-Vortex (WV) separation at the runway threshold (Table 1), which corresponds to the minimal International Civil Aviation Organization (ICAO) separation requirements between two successive landings.

In our MILP formulation, two kinds of decision variables are proposed. First, binary variables are introduced for sequencing purposes as well as for runway and IAF assign-

Table 1. Minimum time separation between two successive landings in seconds, according to the three WV aircraft categories: Heavy (H), Medium (M), and Light (L). Source: Balakrishnan and Chandran (2006)

		Following aircraft		
		H	**M**	**L**
	H	96	157	196
Leading aircraft	**M**	60	69	131
	L	60	69	82

ments. Continuous optimization variables for assigning times at the IAF and at the runway threshold for each aircraft. The objective function is the total schedule delay to be minimized.

4 Preliminary results

The proposed model is implemented and solved via DoCplex, the Python API of CPLEX solver. The test problem instances are generated from real traffic in Paris-Orly airport. These instances feature three IAFs, and two runways (denoted R_1 and R_2 in Table 2).

We compare the three following solutions:

- FCFS: The basic technique usually used by air traffic controllers. It consists in assigning aircraft to the closest IAF according to its origin airport, then to the closest runway from the IAF. The sequence of aircraft is defined based on the First-Come First-Served (FCFS) order.
- RAS-MILP: The runway assignment and sequencing using our MILP model, without IAF assignment decisions.
- RIAS-MILP: The runway and IAF assignment and sequencing using our MILP model.

The results are reported in Table 2 in terms of average delay per aircraft (Avg Delay) of each schedule; it is computed as follows. The set $\mathcal{F}$ denotes the set of aircraft in the schedule, T_f and t_f are respectively the target and the scheduled time of aircraft $f \in \mathcal{F}$. The average delay of the schedule is then given by Equation (1).

$$\text{Avg Delay} = \frac{\sum\limits_{f \in \mathcal{F}} (t_f - T_f)}{|\mathcal{F}|} \tag{1}$$

The results of Table 2 show to what extent an optimization approach can be beneficial for realistic instances of the problem of sequencing and scheduling aircraft, even in the case RAS-MIP – in which IAF assignment is fixed according to the airport of origin – compared to the traditional solution used by controllers (FCFS). Moreover, our complete MILP (RIAS-MILP) yields average delays that are, as expected, further reduced (more degrees of freedom).

The gain remains moderate for these instances, but we are currently working on more congested problems for which our approach is likely to be more valuable.

5 Conclusion and perspectives

This work focuses on the problem of sequencing and scheduling aircraft arrivals at critical terminal airspace fixes and at the runway threshold. We propose a Mixed Integer

Table 2. Average delay comparison between three solution techniques (in seconds), for 10 problem instances from 12:00 am to 10:00 pm

		FCFS			RAS-MILP			RIAS-MILP		
Time	Nb Flights	R_1	R_2	Avg Delay	R_1	R_2	Avg Delay	R_1	R_2	Avg Delay
12:00-13:00	28	18	10	53	12	16	23	15	13	17
13:00-14:00	20	9	11	57	7	13	14	12	8	6
14:00-15:00	15	6	9	60	6	9	20	8	7	13
15:00-16:00	17	11	6	58	8	9	18	10	7	8
16:00-17:00	22	11	11	100	9	13	52	10	12	25
17:00-18:00	21	9	12	64	8	13	17	12	9	0
18:00-19:00	19	9	10	58	8	11	15	13	6	10
19:00-20:00	22	13	9	47	10	12	17	13	9	6
20:00-21:00	18	8	10	62	8	10	24	13	5	4
21:00-22:00	26	11	15	64	14	12	24	15	11	11
Min	15	6	6	53	6	9	14	8	5	0
Max	28	18	15	100	14	16	52	15	13	25

Linear Programming approach that takes into account safety requirements, and maximum acceptable delay based on fuel considerations. The preliminary results show that the average delay can be reduced using our model, compared to the traditional technique used by controllers. In future studies, we plan to validate our model on more congested data from other airports, such as the Paris Charles-de-Gaulle Airport. Then, we aim at discussing further with air traffic controllers so as to improve our model, to render it more realistic, in order to evaluate the viability of our approach in a real-time application.

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